Lesson 12: Chain Rule and the Derivative of $\ln(x)$

Rule $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

Recall
- $\ln(a^n) = n\ln(a)$
- $\ln(ab) = \ln(a) + \ln(b)$
- $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

} use to simplify before taking derivative!

Ex 1 $\frac{d}{dx}[\ln(18x)] = \frac{1}{18x} \cdot \frac{d}{dx}[18x]$ OUT = $\ln(u)$

$= \frac{1}{18x} \cdot 18$

$= \boxed{\frac{1}{x}}$

OR $\frac{d}{dx}[\ln(18x)] = \frac{d}{dx}[\ln(18) + \ln(x)]$

$= 0 + \frac{1}{x}$ ($\ln(18)$ is a constant!)

Ex 2 $y = \ln(t^2 + 7t + e^t)$ OUT = $\ln u$

$y' = \frac{1}{t^2+7t+e^t} \cdot \frac{d}{dt}[t^2+7t+e^t]$

$= \frac{1}{t^2+7t+e^t}(2t+7+e^t)$

$= \boxed{\frac{2t+7+e^t}{t^2+7t+e^t}}$

Ex 3 Simplify $y = \ln(\sqrt[3]{x\cos x})$

$= \ln((x\cos x)^{1/3})$

$= \frac{1}{3}\ln(x\cos x)$

$= \frac{1}{3}(\ln x + \ln(\cos x))$

$(y' = \frac{1}{3x} - \frac{1}{3}\tan x)$

Switching back to chain rule (& everything else!)

Ex 4 $y = x\sec(x^2)$ big idea: Product

$y' = 1 \cdot \sec(x^2) + x \cdot \frac{d}{dx}[\sec(x^2)]$

$= \sec(x^2) + x \frac{d}{dx}[\sec(x^2)]$ Chain! OUT = sec u

$= \sec(x^2) + x\left(\sec(x^2)\tan(x^2) \cdot \frac{d}{dx}[x^2]\right)$

Ex 5 $y = \frac{e^{2x}}{x\sin x}$ big idea: Quotient

$y' = \frac{(x\sin x)\frac{d}{dx}[e^{2x}] - e^{2x}\frac{d}{dx}[x\sin x]}{(x\sin x)^2}$

$\frac{d}{dx}[e^{2x}] \to 2e^{2x}$ (chain)

$\frac{d}{dx}[x\sin x] \to \sin x + x\cos x$ (product)

Ex 6 $y = \tan(t^2 - \cos(2t+7))$ big idea: Chain OUT = tan u

$y' = \sec^2(t^2 - \cos(2t+7))\frac{d}{dt}[t^2 - \cos(2t+7)]$

$= \sec^2(t^2 - \cos(2t+7))\frac{d}{dt}[t^2 - \cos(2t+7)]$ → Chain! OUT = cos u

$= \sec^2(t^2 - \cos(2t+7))\left(2t - \left(-\sin(2t+7)\frac{d}{dt}(2t+7)\right)\right)$

$= \sec^2(t^2 - \cos(2t+7))(2t + 2\sin(2t+7))$

Ex 7 $g(x) = (3x+1)^3(x^2-1)^4$ big idea: Product

$g'(x) = \frac{d}{dx}[(3x+1)^3](x^2-1)^4 + (3x+1)^3\frac{d}{dx}[(x^2-1)^4]$

$= 3(3x+1)^2 \cdot 3(x^2-1)^4 + (3x+1)^3 \cdot 4(x^2-1)^3 \cdot 2x$

$= (3x+1)^2(x^2-1)^3[9(x^2-1) + (3x+1)\cdot 2x]$